PSI–PR–05–01
hep-ph/0501164

Associated Higgs Boson Production with heavy Quarks in e^+e^- Collisions: SUSY–QCD Corrections*

Petra Häfliger[1,2] and Michael Spira[2]

[1] *Institute for Particle Physics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*

Abstract

The processes $e^+e^- \to t\bar{t}/b\bar{b}+$Higgs allow to measure the Yukawa couplings between Higgs bosons and heavy quarks in supersymmetric theories. The complete set of next-to-leading order SUSY–QCD corrections to the cross sections of these processes have been determined in the minimal supersymmetric extension of the Standard Model. They turn out to be $\mathcal{O}(10-20\%)$ and thus important for future linear e^+e^- colliders.

*Supported in part by the Swiss Bundesamt für Bildung und Wissenschaft.

1 Introduction

The Higgs mechanism is a cornerstone of the Standard Model (SM) and its supersymmetric extensions [1]. In the minimal supersymmetric extension of the SM (MSSM) two isospin Higgs doublets have to be introduced in order to generate masses of up- and down-type fermions [2]. After electroweak symmetry breaking three of the eight degrees of freedom are absorbed by the Z and W gauge bosons, implying the existence of five elementary Higgs particles. These consist of two CP-even neutral (scalar) particles h, H, one CP-odd neutral (pseudoscalar) particle A and two charged bosons $H^{\pm}$. At leading order the MSSM Higgs sector is fixed by two independent input parameters which are usually chosen to be the pseudoscalar Higgs mass M_A and $\mathrm{tg}\beta = v_2/v_1$, the ratio of the two vacuum expectation values. Including the one-loop and dominant two-loop corrections the upper bound of the light scalar Higgs mass is $M_h \lesssim 135$ GeV [3]. The couplings of the various neutral Higgs bosons to fermions and gauge bosons, normalized to the SM Higgs couplings, are listed in Table 1, where the angle α denotes the mixing angle of the scalar Higgs bosons h, H. An important property of the bottom Yukawa couplings is their enhancement for large values of $\mathrm{tg}\beta$, while the top Yukawa couplings are suppressed for large $\mathrm{tg}\beta$. This implies that Higgs radiation off bottom quarks plays a significant role at linear e^+e^- colliders in the large $\mathrm{tg}\beta$ regime, while Higgs radiation off top quarks is relevant for small and moderate values of $\mathrm{tg}\beta$.

ϕ		g_u^ϕ	g_d^ϕ	g_V^ϕ
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\mathrm{tg}\beta$	$\mathrm{tg}\beta$	0

Table 1: *Higgs couplings in the MSSM to fermions and gauge bosons [$V = W, Z$] relative to the SM couplings.*

For the computation of the SUSY-QCD corrections we need the Higgs couplings to stop and sbottom squarks in addition, as well as the squark couplings to the photon and Z boson. The scalar superpartners $\tilde{f}_{L,R}$ of the left- and right-handed fermion components mix with each other. The mass eigenstates $\tilde{f}_{1,2}$ of the sfermions are related to the current eigenstates $\tilde{f}_{L,R}$ by mixing angles θ_f,

$$\begin{aligned} \tilde{f}_1 &= \tilde{f}_L \cos\theta_f + \tilde{f}_R \sin\theta_f \\ \tilde{f}_2 &= -\tilde{f}_L \sin\theta_f + \tilde{f}_R \cos\theta_f \,. \end{aligned} \tag{1}$$

The mass matrix of the sfermions in the left-right-basis is given by [4][1]

$$\mathcal{M}_{\tilde{f}} = \begin{bmatrix} M^2_{\tilde{f}_L} + m_f^2 & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & M^2_{\tilde{f}_R} + m_f^2 \end{bmatrix}, \tag{2}$$

with the parameters $r_d = 1/r_u = \mathrm{tg}\beta$. The parameters A_f denote the trilinear scalar couplings of the soft supersymmetry breaking part of the Lagrangian. The mixing angles acquire the form

$$\sin 2\theta_f = \frac{2m_f(A_f - \mu r_f)}{M^2_{\tilde{f}_1} - M^2_{\tilde{f}_2}} \quad , \quad \cos 2\theta_f = \frac{M^2_{\tilde{f}_L} - M^2_{\tilde{f}_R}}{M^2_{\tilde{f}_1} - M^2_{\tilde{f}_2}}, \tag{3}$$

and the masses of the squark mass eigenstates are given by

$$M^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[M^2_{\tilde{f}_L} + M^2_{\tilde{f}_R} \mp \sqrt{(M^2_{\tilde{f}_L} - M^2_{\tilde{f}_R})^2 + 4m_f^2(A_f - \mu r_f)^2}\right]. \tag{4}$$

Since the mixing angles are proportional to the masses of the ordinary fermions, mixing effects are only important for the third-generation sfermions. The neutral Higgs couplings to sfermions read [5]

$$\begin{aligned} g^\phi_{\tilde{f}_L\tilde{f}_L} &= m_f^2 g_1^\phi + M_Z^2(I_{3f} - e_f \sin^2\theta_W) g_2^\phi \\ g^\phi_{\tilde{f}_R\tilde{f}_R} &= m_f^2 g_1^\phi + M_Z^2 e_f \sin^2\theta_W g_2^\phi \\ g^\phi_{\tilde{f}_L\tilde{f}_R} &= -\frac{m_f}{2}(\mu g_3^\phi - A_f g_4^\phi), \end{aligned} \tag{5}$$

with the couplings g_i^ϕ listed in Table 2. I_{3f} denotes the third component of the electroweak isospin, e_f the electric charge of the fermion f and $\sin\theta_W$ the Weinberg angle, m_f the fermion mass and M_Z the Z-boson mass.

Finally the sfermion couplings to photons are given by [5]

$$g^\gamma_{\tilde{f}_i\tilde{f}_j} = e e_f \delta_{ij} \tag{6}$$

and to Z bosons by

$$\begin{aligned} g^Z_{\tilde{f}_L\tilde{f}_L} &= \frac{e}{s_W c_W}(I_{3f} - e_f \sin^2\theta_W) \\ g^Z_{\tilde{f}_R\tilde{f}_R} &= -\frac{e}{s_W c_W} e_f \sin^2\theta_W \\ g^Z_{\tilde{f}_L\tilde{f}_R} &= 0\,. \end{aligned} \tag{7}$$

All these couplings have to be rotated to the mass eigenstates by the mixing angle θ_f. The coupling $e = \sqrt{4\pi\alpha}$ denotes the elementary electric charge. The Yukawa couplings

[1] For simplicity, the D-terms have been absorbed in the sfermion-mass parameters $M^2_{\tilde{f}_{L/R}}$.

$\tilde{f}$	ϕ	g_1^ϕ	g_2^ϕ	g_3^ϕ	g_4^ϕ
	h	$\cos\alpha/\sin\beta$	$-\sin(\alpha+\beta)$	$-\sin\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
$\tilde{u}$	H	$\sin\alpha/\sin\beta$	$\cos(\alpha+\beta)$	$\cos\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
	A	0	0	-1	$1/\mathrm{tg}\beta$
	h	$-\sin\alpha/\cos\beta$	$-\sin(\alpha+\beta)$	$\cos\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$
$\tilde{d}$	H	$\cos\alpha/\cos\beta$	$\cos(\alpha+\beta)$	$\sin\alpha/\cos\beta$	$\cos\alpha/\cos\beta$
	A	0	0	-1	$\mathrm{tg}\beta$

Table 2: *Coefficients of the neutral MSSM Higgs couplings to sfermion pairs.*

of squarks to gluinos and quarks are uniquely determined by the strong coupling α_s. The corresponding Feynman rules can be found in Ref. [4].

Neutral Higgs radiation off top or bottom quarks $[Q = t, b]$ in e^+e^- collisions,

$$e^+e^- \to Q\bar{Q}\phi^0 \qquad [\phi^0 = h, H, A], \tag{8}$$

is a suitable process for measuring the Yukawa couplings in supersymmetric theories [6], particularly for the light Higgs boson h and for moderately heavy Higgs bosons H and A. In the following we present the cross sections for these processes including the next-to-leading order (NLO) SUSY-QCD corrections. The pure QCD corrections have been determined before in the SM [7] and the MSSM [8] (approximate results can be found in Refs. [9,10]). They significantly modify the cross section at high collider energies. For SM Higgs boson radiation off top quarks the full electroweak corrections have recently become available [11]. They turn out to be of similar magnitude as the pure QCD corrections so that an analogous result may be expected for MSSM Higgs bosons. First results of the SUSY-QCD corrections to $t\bar{t}h$ production appeared in Ref [12]. We will comment on the large size of these corrections in this paper.

2 Calculation of NLO SUSY-QCD Corrections

In this section we will set up our notation and summarize the present status of the processes under investigation, before we will describe the novel SUSY-QCD corrections in detail.

2.1 Leading-Order Cross Sections and QCD Corrections

At leading order (LO) Higgs radiation off heavy quarks is described by the Feynman diagrams of Fig. 1. It splits into three different classes of contributions: (i) Higgs radiation

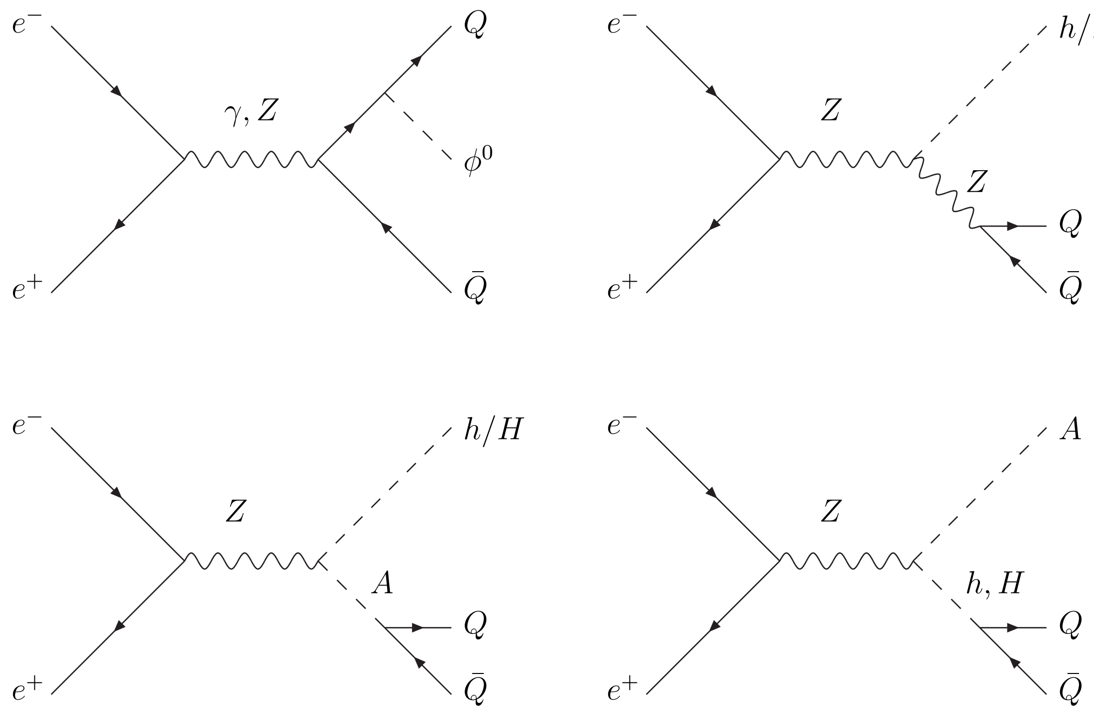


Figure 1: *Individual channels of scalar and pseudoscalar Higgs radiation off heavy quarks $Q = t, b$ in e^+e^- collisions.*

off the heavy (anti)quark, (ii) Higgs radiation off the Z boson (only scalar Higgs radiation) and (iii) Z boson splitting into scalar-pseudoscalar Higgs pairs with one of them dissociating into a heavy $Q\bar{Q}$ pair. Depending on the masses of the corresponding particles resonant contributions will arise, which require the inclusion of finite decay widths of the Z and Higgs bosons in the corresponding propagators. We have used conventional Breit-Wigner propagators for the resonant $Z \to b\bar{b}$ and $\phi^0 \to t\bar{t}/b\bar{b}$ decays as in previous analyses [8]. The LO matrix elements can be decomposed according to the different Higgs couplings,

$$\begin{aligned} \mathcal{M}_{LO}^{h/H} &= g_Q^{h/H}\mathcal{C}_1^{LO} + g_Z^{h/H}\mathcal{C}_2^{LO} + g_Q^A\mathcal{C}_3^{LO} \\ \mathcal{M}_{LO}^{A} &= g_Q^A\mathcal{D}_1^{LO} + g_Q^h\mathcal{D}_2^{LO} + g_Q^H\mathcal{D}_3^{LO} \,. \end{aligned} \tag{9}$$

The LO cross sections can be cast into the form

$$\sigma_{LO} = \int dPS_3 \sum_{spins,colours} \overline{|\mathcal{M}_{LO}^{\phi^0}|^2} \tag{10}$$

with dPS_3 denoting the three-particle phase space element. The sum has to be taken over the spins of the initial and final-state fermions and colours supplemented by an average over the initial e^+e^- spins.

We include the QCD corrections of Ref. [8] with the QCD coupling α_s evaluated at NLO with 5 active flavours at the renormalization scale $\mu_R = \sqrt{s}$ with s being the e^+e^-

c.m. energy squared. The coupling α_s is normalized to $\alpha_s(M_Z^2) = 0.119$. The bottom Yukawa couplings are computed at the scale of the corresponding Higgs-momentum flow. This choice absorbs large logarithmic contributions of the pure QCD corrections [8].

The value of the electromagnetic coupling is taken to be $\alpha = 1/128$ and the Weinberg angle to be $s_W^2 = 0.23$. The mass of the Z boson is set to $M_Z = 91.187$ GeV, and the pole masses of the top and bottom quarks are set to $m_t = 175$ GeV[2] and $m_b = 4.62$ GeV[3], respectively [8]. The masses of the MSSM Higgs bosons and their couplings are related to tgβ and the pseudoscalar Higgs-boson mass M_A. In the relations we use, higher-order corrections up to two loops in the effective-potential approach are included [14]. The Z boson width has been chosen as $\Gamma_Z = 2.49$ GeV, and the Higgs boson widths have been computed with the program HDECAY [15].

2.2 SUSY-QCD Corrections

The NLO SUSY-QCD corrections arise from virtual gluino and stop/sbottom exchange contributions as depicted in Fig. 2. They consist of self-energy, vertex and box contribu-

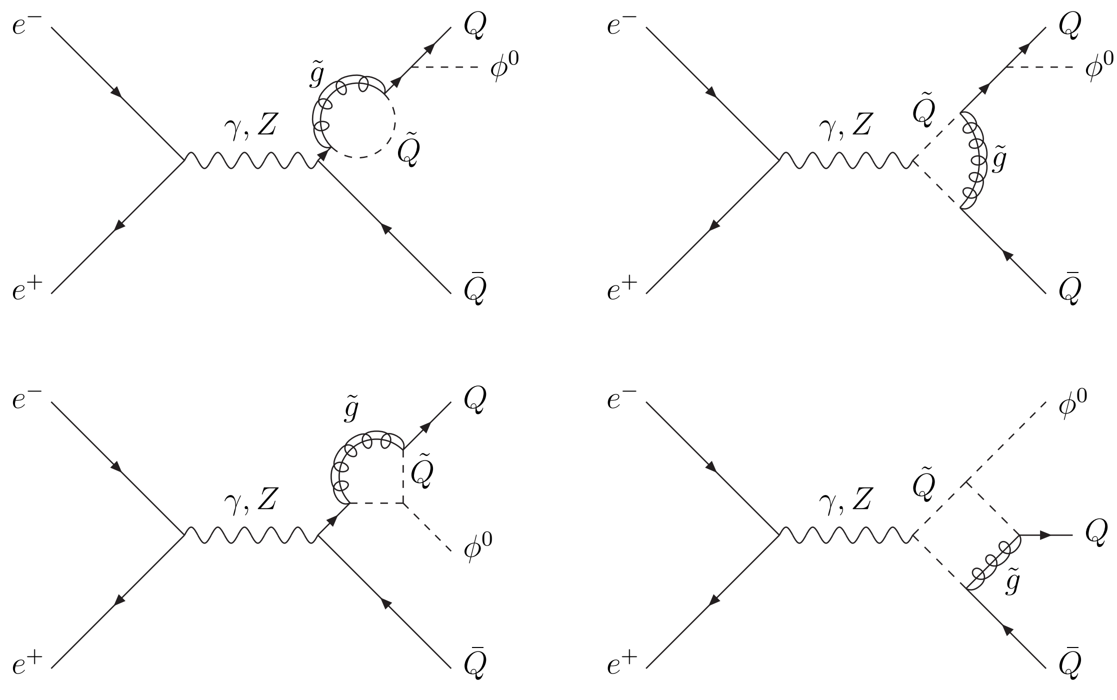


Figure 2: *Typical diagrams of the NLO SUSY-QCD corrections to* $e^+e^- \to Q\bar{Q}\phi^0$ *[*$Q = t, b$*] mediated by gluino* $\tilde{g}$ *and squark* $\tilde{Q} = \tilde{t}, \tilde{b}$ *exchange.*

[2]The top mass has been chosen in accordance with the definitions of the Snowmass-benchmark points of the MSSM [13].

[3]This value for the perturbative pole mass of the bottom quark corresponds in NLO to an $\overline{\mathrm{MS}}$ mass $\overline{m}_b(\overline{m}_b) = 4.28$ GeV.

tions, which are calculated within dimensional regularization in the standard way. Since all virtual particles are massive, no infrared nor collinear singularities arise. The ultraviolet divergences are removed by the renormalization of the quark masses and Yukawa couplings. The latter is connected to the quark mass renormalization. In the case of $t\bar{t}$+Higgs production the top mass has been renormalized on-shell in the propagators as well as in the Yukawa couplings. The same prescription has also been chosen for the bottom mass, since the virtual gluino and sbottom masses are too large to develop large logarithmic contributions. Thus the renormalization of the bottom quark mass is given by

$$\begin{aligned} m_b^0 &= \overline{m}_b(\mu^2)\left[1+\delta_{QCD}+\delta_{SQCD}\right] \\ \delta_{QCD} &= -\frac{\alpha_s}{\pi}\Gamma(\epsilon)(4\pi)^\epsilon \\ \delta_{SQCD} &= \frac{\tilde{\Sigma}(m_b)}{\overline{m}_b(\mu^2)}\,, \end{aligned} \tag{11}$$

where m_b^0 denotes the bare bottom mass, $\overline{m}_b(\mu^2)$ the $\overline{\mathrm{MS}}$ mass at the scale μ and $\delta_{(S)QCD}$ the corresponding (SUSY-)QCD counter terms. The SUSY-QCD contribution to the bottom quark self-energy is displayed by $\tilde{\Sigma}(m_b)$ with on-shell momentum. This renormalization prescription ensures that the gluino and sbottom contributions are decoupled from the running of the bottom Yukawa couplings. Thus we are left with the pure $\overline{\mathrm{MS}}$ Yukawa couplings of QCD.

The final result can be cast into the form

$$\sigma(e^+e^-\to Q\bar{Q}\phi^0) = \sigma_{LO}(e^+e^-\to Q\bar{Q}\phi^0)\left\{1+\left[C_{QCD}+C_{SQCD}\right]\frac{\alpha_s(s)}{\pi}\right\}\,, \tag{12}$$

where $\sigma_{LO}(e^+e^-\to Q\bar{Q}\phi^0)$ denotes the LO cross section and $C_{(S)QCD}$ the coefficients of the (SUSY-)QCD corrections.

For large values of tgβ there are significant non-decoupling corrections to $b\bar{b}\phi^0$ production, which can be absorbed in the bottom Yukawa couplings in a universal way [16][4]. In Refs. [18,19] it has been shown that these contributions can be resummed to improve the reliability of the perturbative result. The resummed bottom Yukawa couplings are given by[5]

$$\begin{aligned} \tilde{g}_b^h &= \frac{g_b^h}{1+\Delta_b}\left(1-\frac{\Delta_b}{\mathrm{tg}\alpha\ \mathrm{tg}\beta}\right) \\ \tilde{g}_b^H &= \frac{g_b^H}{1+\Delta_b}\left(1+\Delta_b\frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}\right) \\ \tilde{g}_b^A &= \frac{g_b^A}{1+\Delta_b}\left(1-\frac{\Delta_b}{\mathrm{tg}^2\beta}\right) \end{aligned} \tag{13}$$

[4] It should be noted that these contributions vanish for large sbottom and gluino masses while keeping the μ parameter fixed [17]. Non-decoupling effects only arise, if the μ parameter is increased together with the SUSY particle masses.

[5] Analogous effective couplings can be defined for top quarks, too, but in this case the non-decoupling contributions are small and thus do not require resummation.

with the quantities [19]

$$\begin{aligned}
\Delta_b &= \frac{\Delta m_b}{1+\Delta_1} \\
\Delta m_b &= \frac{2}{3}\,\frac{\alpha_s}{\pi}\, m_{\tilde g}\;\mu\;\mathrm{tg}\beta\; I(m^2_{\tilde b_1}, m^2_{\tilde b_2}, m^2_{\tilde g}) \\
\Delta_1 &= -\frac{2}{3}\,\frac{\alpha_s}{\pi}\, m_{\tilde g}\; A_b\; I(m^2_{\tilde b_1}, m^2_{\tilde b_2}, m^2_{\tilde g}) \\
I(a,b,c) &= -\frac{ab\log\frac{a}{b} + bc\log\frac{b}{c} + ca\log\frac{c}{a}}{(a-b)(b-c)(c-a)}\,.
\end{aligned} \tag{14}$$

If the LO cross sections are expressed in terms of these resummed bottom Yukawa couplings, the corresponding NLO pieces have to be subtracted from the coefficient C_{SQCD} to avoid double counting. This is equivalent to an additional (finite) renormalization, given explicitly by

$$\begin{aligned}
g_b^\phi &\to \tilde g_b^\phi \left[1+\Delta_b^\phi\right] + \mathcal{O}(\alpha_s^2) \\
\Delta_b^\phi &= \frac{2}{3}\,\frac{\alpha_s}{\pi}\,\kappa_\phi\; m_{\tilde g}\;\mu\;\mathrm{tg}\beta\; I(m^2_{\tilde b_1}, m^2_{\tilde b_2}, m^2_{\tilde g}) \\
\kappa_h &= 1+\frac{1}{\mathrm{tg}\alpha\;\mathrm{tg}\beta} \\
\kappa_H &= 1-\frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta} \\
\kappa_A &= 1+\frac{1}{\mathrm{tg}^2\beta}\,.
\end{aligned} \tag{15}$$

Thus we have to add additional finite counter terms to the virtual matrix elements[6]

$$\begin{aligned}
\Delta\mathcal{M}^{h/H}_{SQCD} &= g_b^{h/H}\Delta_b^{h/H}\mathcal{C}_1 + g_b^A\Delta_b^A\mathcal{C}_3 \\
\Delta\mathcal{M}^{A}_{SQCD} &= g_b^{A}\Delta_b^{A}\mathcal{D}_1 + g_b^h\Delta_b^h\mathcal{D}_2 + g_b^H\Delta_b^H\mathcal{D}_3\,.
\end{aligned} \tag{16}$$

In the LO matrix elements we use the resummed Yukawa couplings,

$$\begin{aligned}
\widetilde{\mathcal{M}}^{h/H}_{LO} &= \tilde g_b^{h/H}\mathcal{C}_1^{LO} + g_Z^{h/H}\mathcal{C}_2^{LO} + \tilde g_b^A\mathcal{C}_3^{LO} \\
\widetilde{\mathcal{M}}^{A}_{LO} &= \tilde g_b^{A}\mathcal{D}_1^{LO} + \tilde g_b^h\mathcal{D}_2^{LO} + \tilde g_b^H\mathcal{D}_3^{LO}\,,
\end{aligned} \tag{17}$$

so that the SUSY-QCD corrections to the cross sections are given by

$$\Delta\sigma_{SQCD} = \sigma_{LO} C_{SQCD}\frac{\alpha_s}{\pi} = \int dPS_3\; 2\Re e \sum_{spins,colours} \overline{\widetilde{\mathcal{M}}^{\phi^0\dagger}_{LO}\mathcal{M}^{\phi^0}_{SQCD}}\,. \tag{18}$$

In the QCD corrections we also insert the resummed bottom Yukawa couplings everywhere, since the non-decoupling terms $\Delta_b^{\phi^0}$ factorize from the pure QCD corrections initiated by light particle interactions.

[6]Note that in the matrix elements of the SUSY-QCD corrections we keep the unresummed bottom Yukawa couplings in order to avoid artificial singularities for vanishing mixing angle α [19].

3 Results

The numerical results will be presented for a linear e^+e^- collider with c.m. energy of 1 TeV. We have chosen the Snowmass point SPS5 for Higgs radiation off top quarks and SPS1b for the bottom quark case [13]. The MSSM parameters of these two benchmark scenarios are given by[7]

SPS5:

$$\begin{aligned} \mathrm{tg}\beta &= 5 \\ \mu &= 639.8 \text{ GeV} \\ A_t &= -1671.4 \text{ GeV} \\ A_b &= -905.6 \text{ GeV} \\ m_{\tilde{g}} &= 710.3 \text{ GeV} \\ m_{\tilde{q}_L} &= 535.2 \text{ GeV} \\ m_{\tilde{b}_R} &= 620.5 \text{ GeV} \\ m_{\tilde{t}_R} &= 360.5 \text{ GeV} \end{aligned} \tag{19}$$

SPS1b:

$$\begin{aligned} \mathrm{tg}\beta &= 30 \\ \mu &= 495.6 \text{ GeV} \\ A_t &= -729.3 \text{ GeV} \\ A_b &= -987.4 \text{ GeV} \\ m_{\tilde{g}} &= 916.1 \text{ GeV} \\ m_{\tilde{q}_L} &= 762.5 \text{ GeV} \\ m_{\tilde{b}_R} &= 780.3 \text{ GeV} \\ m_{\tilde{t}_R} &= 670.7 \text{ GeV}\,. \end{aligned} \tag{20}$$

The pseudoscalar Higgs mass is left free in both scenarios in order to scan the corresponding Higgs mass ranges.

The total cross section for pseudoscalar Higgs radiation off top quarks is displayed at LO and NLO in Fig. 3a. The cross section is small for pseudoscalar Higgs masses below about 350 GeV, while above it rapidly increases to a level of 1 fb due to the intermediate on-shell $H \to t\bar{t}$ decay. The total size of the corrections amounts to about $\mathcal{O}(10\%)$ apart from the threshold of the resonant contribution, where the Coulomb singularity raises the QCD corrections to more than 100% [7]. The Coulomb singularity is an artefact of the narrow-width approximation. A proper treatment of the threshold region requires the inclusion of finite-width effects and QCD-potential contributions. Thus the result

[7] We have neglected the corresponding translations of $\overline{\mathrm{DR}}$ masses into $\overline{\mathrm{MS}}$ masses, since they are not relevant for the characterization of the results.

obtained in this work is not valid in a small margin around the $t\bar{t}$ threshold of the resonant part. The individual relative corrections, defined as $\sigma_{NLO} = \sigma_{LO}(1 + \delta_{QCD} + \delta_{SQCD})$, can be inferred from Fig. 3b. Except for the threshold region of the resonant part, the QCD corrections are of moderate size [8]. The SUSY-QCD corrections are of similar magnitude as the pure QCD corrections but of opposite sign. Thus, we observe a large cancellation of the QCD corrections against the SUSY-QCD part. This signalizes the importance of including both types of corrections in future analyses. An analogous picture emerges for the light and heavy scalar Higgs bosons as shown in Figs. 4a and b. The cross section for the light scalar Higgs boson is always of the order of 1 fb with small corrections due to the partial cancellation of QCD and SUSY-QCD corrections (see Fig. 4b). The QCD Coulomb singularity for $M_A \sim 350$ GeV is much more pronounced than in the pseudoscalar case, since for the heavy scalar Higgs boson the S-wave pseudoscalar Higgs decay $A \to t\bar{t}$ constitutes the resonant part[8]. The relative threshold corrections remain finite in both cases due to the remaining continuum contributions.

The results for $b\bar{b}A$ production are presented in Fig. 5. The total cross section, shown in Fig. 5a, reaches a size of $\mathcal{O}(10 \text{ fb})$ for smaller pseudoscalar masses. The relative corrections are depicted in Fig. 5b. The pure SUSY-QCD and total corrections are shown without and with resummation of the Δ_b terms according to Eqs. (13–18). It is clearly visible that the resummed bottom Yukawa couplings absorb the bulk of the SUSY-QCD corrections, so that the terms of Eq. (16) provide a reasonable approximation of the final result. After resummation the SUSY-QCD corrections cancel against the pure QCD corrections to a large extent. Thus, as in the top quark case the inclusion of both corrections is of vital importance. A comparison of the total resummed and unresummed NLO cross sections in Fig. 5a implies good agreement within 10% and thus a significant improvement of the perturbative stability from LO to NLO. An analogous picture emerges for the light and heavy scalar Higgs bosons as can be inferred from Fig. 6. Again the resummed Yukawa couplings absorb the bulk of the SUSY-QCD corrections. A significant cancellation of the QCD and SUSY-QCD corrections is observed after resummation in this case, too. The drops of the relative corrections towards $M_A \sim 500$ GeV in Figs. 5 and 6 are caused by the kinematical closure of the intermediate on-shell HA pair production.

Higgs radiation off bottom quarks is, however, dominated by the resonant $h, H \to b\bar{b}$ decays in the pseudoscalar case and the resonant $Z, A \to b\bar{b}$ decays in the scalar case. Thus, the absorption of the bulk of the SUSY-QCD part by the resummed Yukawa couplings could be expected from the analogous findings for the corresponding Higgs decays [19]. In order to investigate, if this also holds for continuum $b\bar{b}\phi^0$ production, we analyze the Higgs energy distribution in Fig. 7 for pseudoscalar Higgs radiation off bottom quarks for a pseudoscalar Higgs mass $M_A = 200$ GeV. The dimensionless parameters x_{ϕ^0} are defined as $x_{\phi^0} = 2E_{\phi^0}/\sqrt{s}$. Fig. 7a displays the x_A distribution at LO and NLO, while Fig. 7b exhibits the individual relative corrections. The sharp peak at $x_A \sim 1$ originates from the resonant $h, H \to b\bar{b}$ decays, while the regions apart from the peak represent

[8]Since the pseudoscalar Higgs boson A carries the same quantum numbers as the 0^{-+} ground state of the $t\bar{t}$ pair, the decay $A \to t\bar{t}$ is dominated by an S-wave contribution at threshold. In contrast the scalar Higgs decay $H \to t\bar{t}$ suffers from a P-wave suppression at threshold.

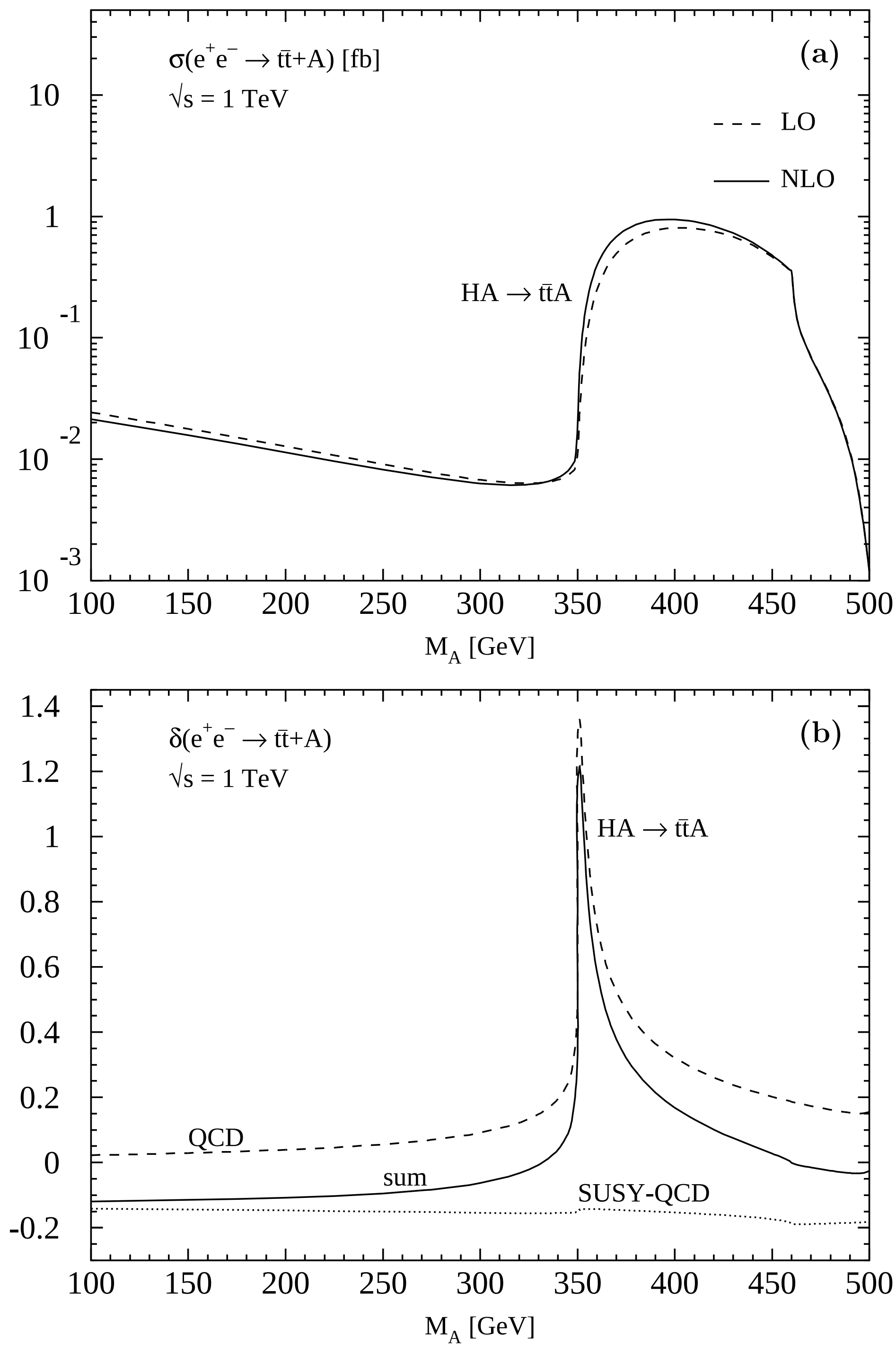


Figure 3: *(a) Production cross sections of pseudoscalar Higgs radiation off top quarks in e^+e^- collisions. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line; (b) Relative QCD, SUSY-QCD and total corrections to pseudoscalar Higgs radiation off top quarks. The sharp (finite) peak around $M_A = 350$ GeV originates from the Coulomb singularity in the QCD corrections to the resonant $H \to t\bar{t}$ decay.*

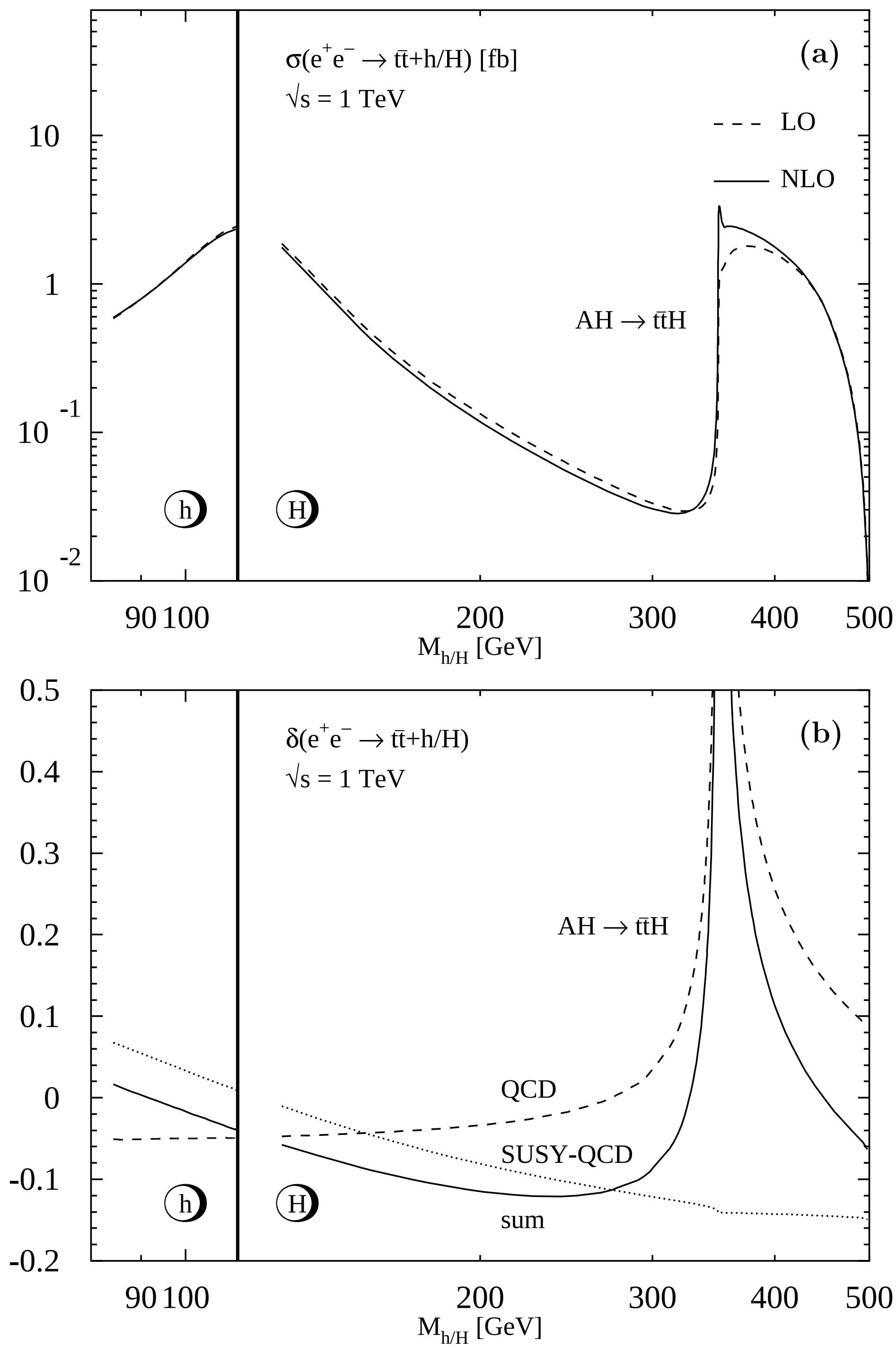


Figure 4: *(a) Production cross sections of heavy and light scalar Higgs radiation off top quarks in e^+e^- collisions. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line; (b) Relative QCD, SUSY-QCD and total corrections to scalar Higgs radiation off top quarks. The sharp (finite) peak around $M_H = 350$ GeV originates from the Coulomb singularity in the QCD corrections to the resonant $A \to t\bar{t}$ decay.*

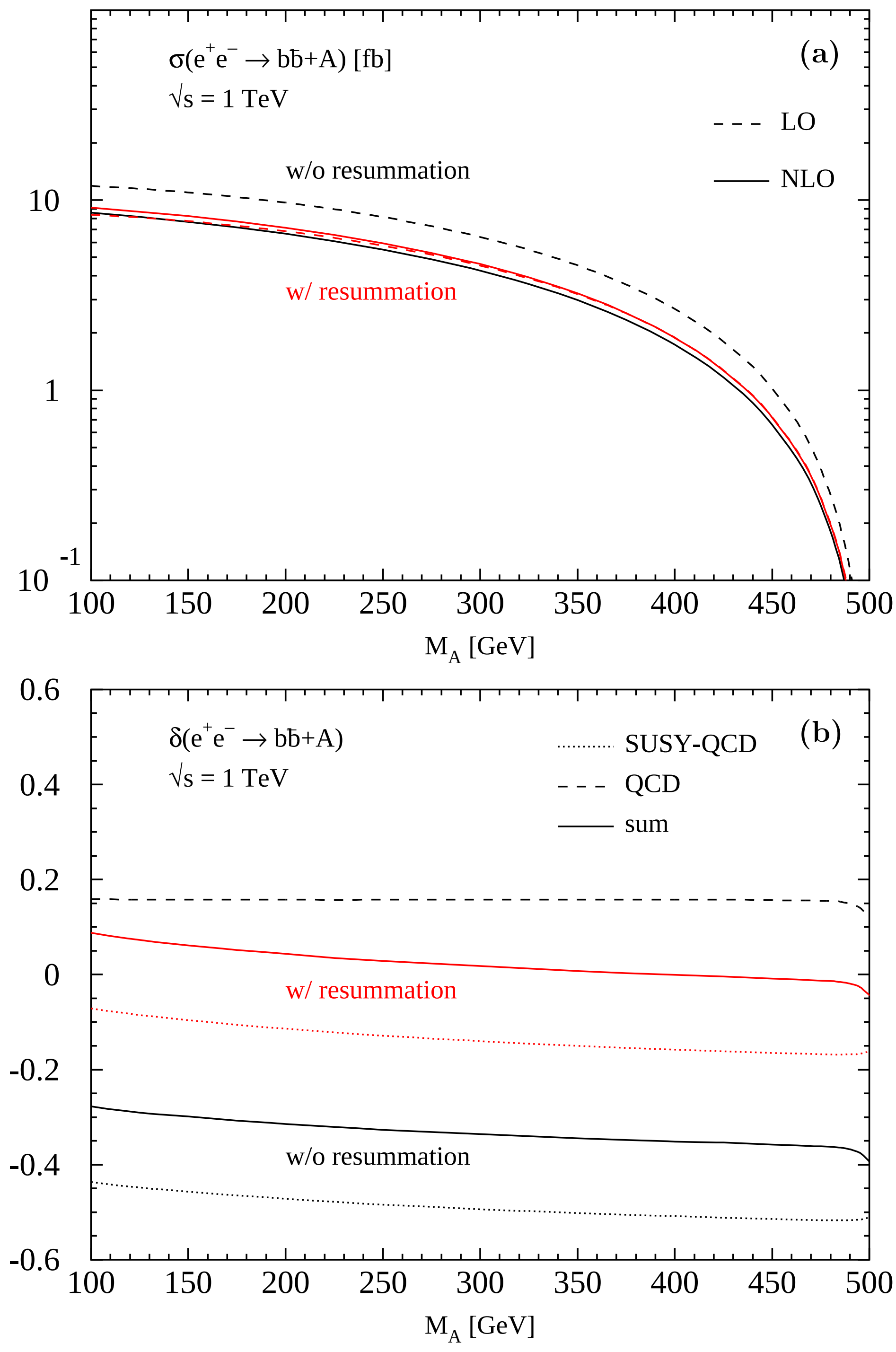


Figure 5: *(a) Production cross sections of pseudoscalar Higgs radiation off bottom quarks in e^+e^- collisions with (red curves) and without (black curves) resummation of the Δ_b terms. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line; (b) Relative QCD, SUSY-QCD and total corrections to pseudoscalar Higgs radiation off bottom quarks with (red lines) and without (black lines) resummation. The pure QCD corrections are indistinguishable in both cases.*

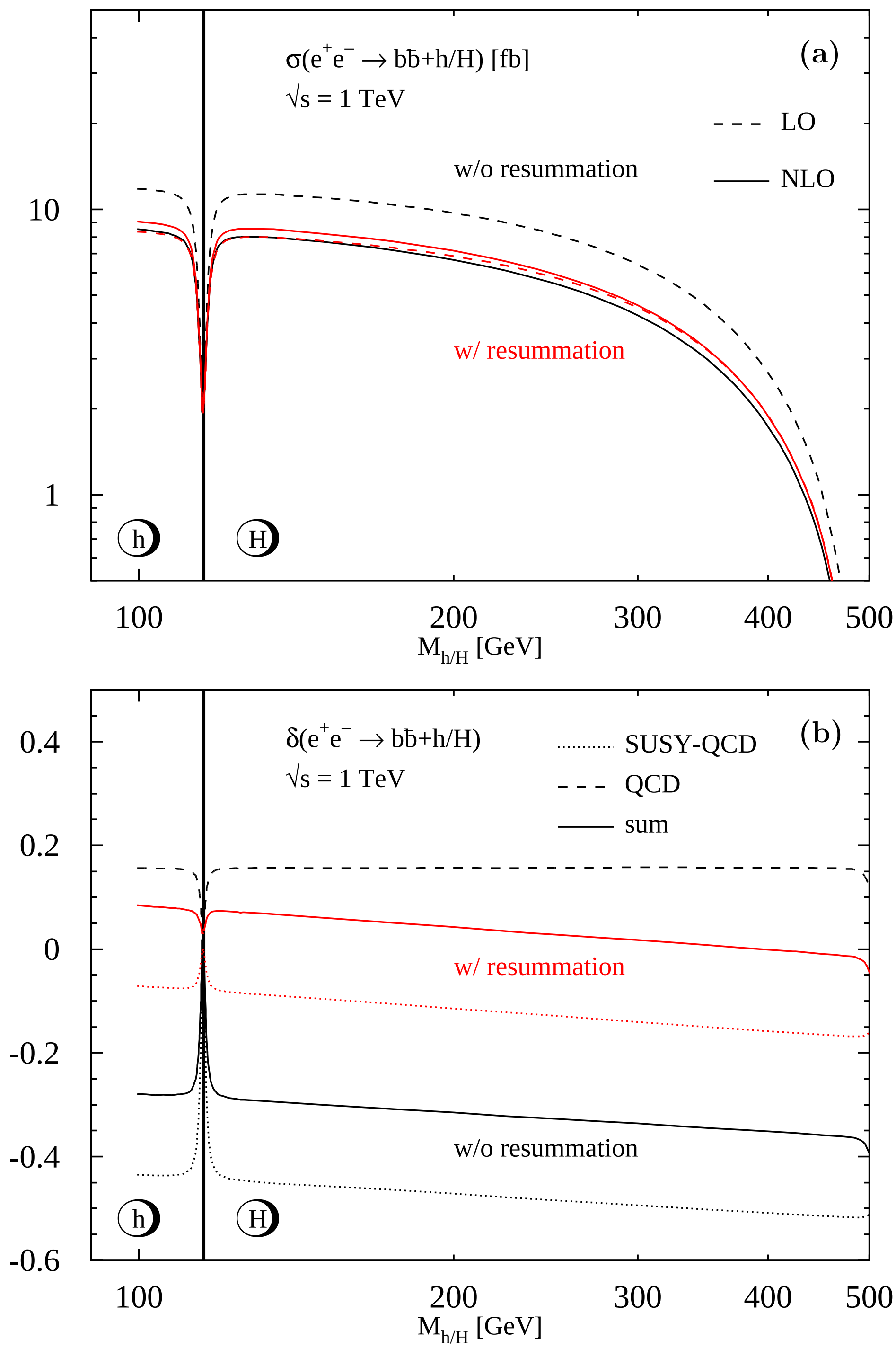


Figure 6: *(a) Production cross sections of heavy and light scalar Higgs radiation off bottom quarks in e^+e^- collisions with (red curves) and without (black curves) resummation of the Δ_b terms. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line; (b) Relative QCD, SUSY-QCD and total corrections to scalar Higgs radiation off bottom quarks with (red lines) and without (black lines) resummation. The pure QCD corrections are indistinguishable in both cases.*

continuum $b\bar{b}A$ production. The resulting picture indeed turns out to be analogous to the total cross sections. The bulk of the SUSY-QCD corrections can be absorbed by the resummed bottom Yukawa couplings leaving moderate residual corrections. These cancel the pure QCD corrections to a large extent in the resonant as well as the continuum regions.

The light scalar Higgs energy distribution for Higgs radiation off top quarks is shown in Fig. 8 for a light scalar Higgs mass $M_h = 100$ GeV. For $x_h \lesssim 0.8$ both the QCD and SUSY-QCD corrections are of moderate size and cancel each other partly. The sum of the corrections amounts to a few per cent. The sharp rise of the QCD corrections towards $x_h \sim 0.9$ is induced by the Coulomb singularity at the subthreshold of the $t\bar{t}$ pair [8]. It leads to a finite cross section at the upper bound of the x_h range. Since the total corrections are not constant, the shape of the Higgs energy distribution is slightly modified from LO to NLO as can be inferred from Fig. 8a.

We performed a comparison of our $t\bar{t}h$ results with those of Ref. [12]. We could not tune our MSSM scenario exactly to the one of Ref. [12], since we do not use the same MSSM routines to calculate the SUSY-particle masses and couplings. Nevertheless, we have found reasonable agreement with the results in a scenario very close to the one in Ref. [12]. However, this scenario is characterized by a very extreme choice of parameters. The light stop $\tilde{t}_1$ is very light ($m_{\tilde{t}_1} \sim 100$ GeV), and the gluino mass of 200 GeV is chosen such that $m_t + M_h \sim m_{\tilde{g}} + m_{\tilde{t}_1}$, so that the thresholds of the virtual Yukawa vertex corrections in the momentum of the off-shell (anti)top quarks, which split into $t/\bar{t}+h$, are very close to the average momentum flow through the (anti)top propagator (see the third diagram of Fig. 2). This enhances the size of the SUSY-QCD corrections considerably. However, the theoretical uncertainties in the threshold regions are large, and the results cannot be trusted quantitatively. A better perturbative treatment requires the inclusion of finite-width effects of the unstable particles as well as QCD-potential effects. Since the scenario of Ref. [12] belongs to a very particular region in the MSSM parameter space, this result does not represent the overall size of the corrections. It can clearly be inferred from Fig. 6 of Ref. [12] that the corrections are of moderate size away from this threshold region.

4 Conclusions

We have presented the full SUSY-QCD corrections to neutral MSSM Higgs radiation off top and bottom quarks at linear e^+e^- colliders. The size of the corrections is of $\mathcal{O}(10-20\%)$ and of similar magnitude as the pure QCD corrections obtained in the past, but of opposite sign, so that significant cancellations occur. This underlines the relevance of including these corrections in future analyses of these processes at linear e^+e^- colliders.

At large values of tgβ Higgs radiation off bottom quarks provides a possibility to measure tgβ [20]. In the past it has been demonstrated that the bulk of the pure QCD corrections can be absorbed in the running bottom Yukawa couplings, defined at the scale of the corresponding Higgs momentum flows [8]. The SUSY-QCD corrections on

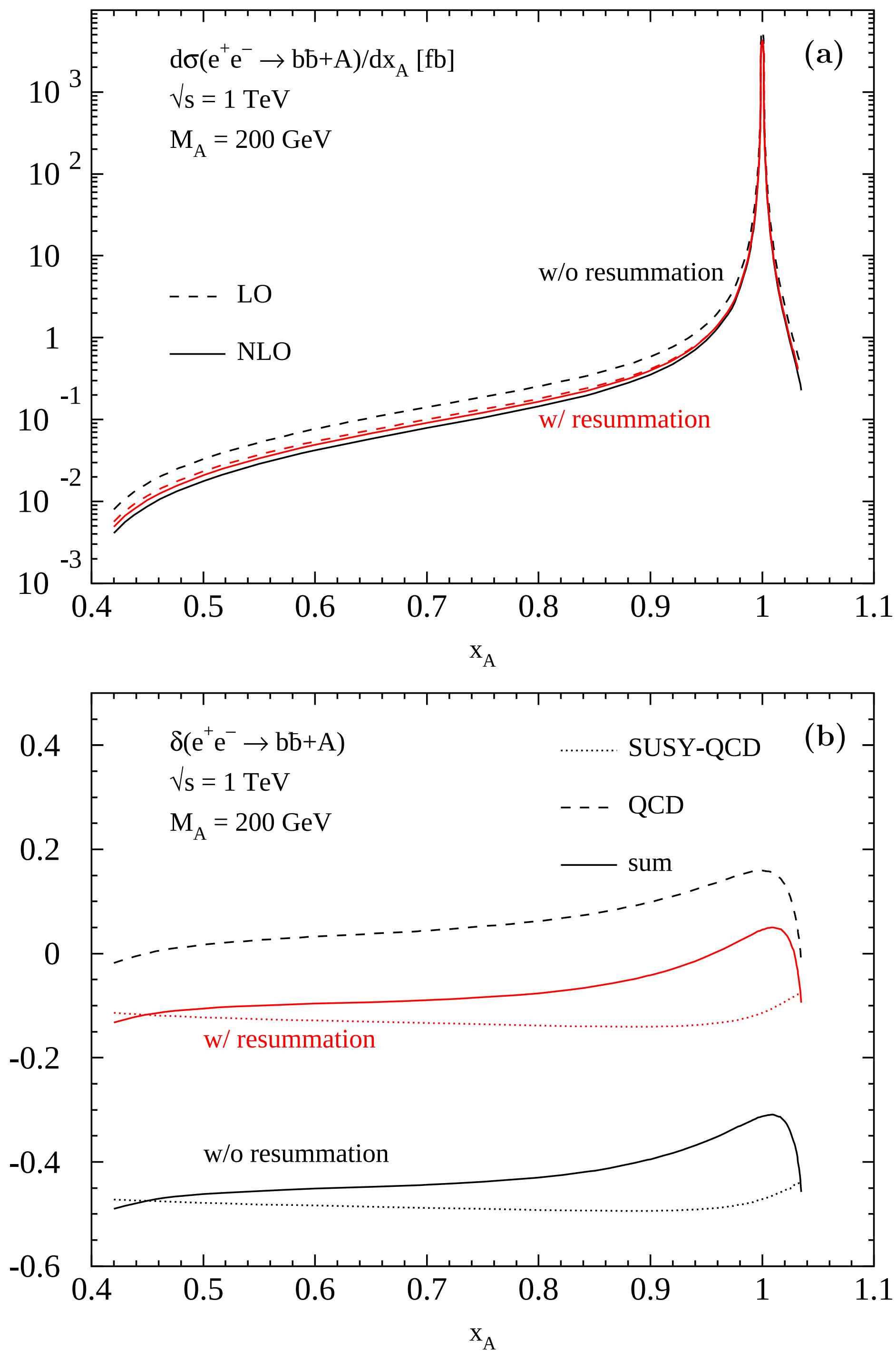
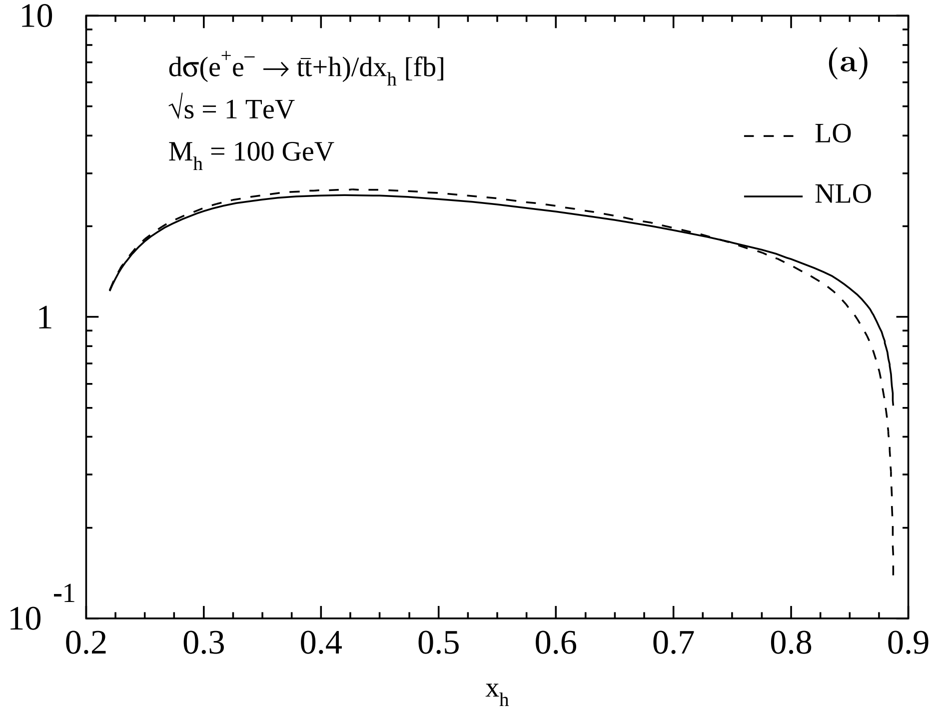


Figure 7: *(a) Higgs energy distribution of pseudoscalar Higgs radiation off bottom quarks in e^+e^- collisions with (red curves) and without (black curves) resummation of the Δ_b terms. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line. The peak at $x_A \sim 1$ originates from the resonant $h, H \to b\bar{b}$ decays; (b) Relative QCD, SUSY-QCD and total corrections to pseudoscalar Higgs radiation off bottom quarks with (red lines) and without (black lines) resummation. The pure QCD corrections are indistinguishable in both cases.*



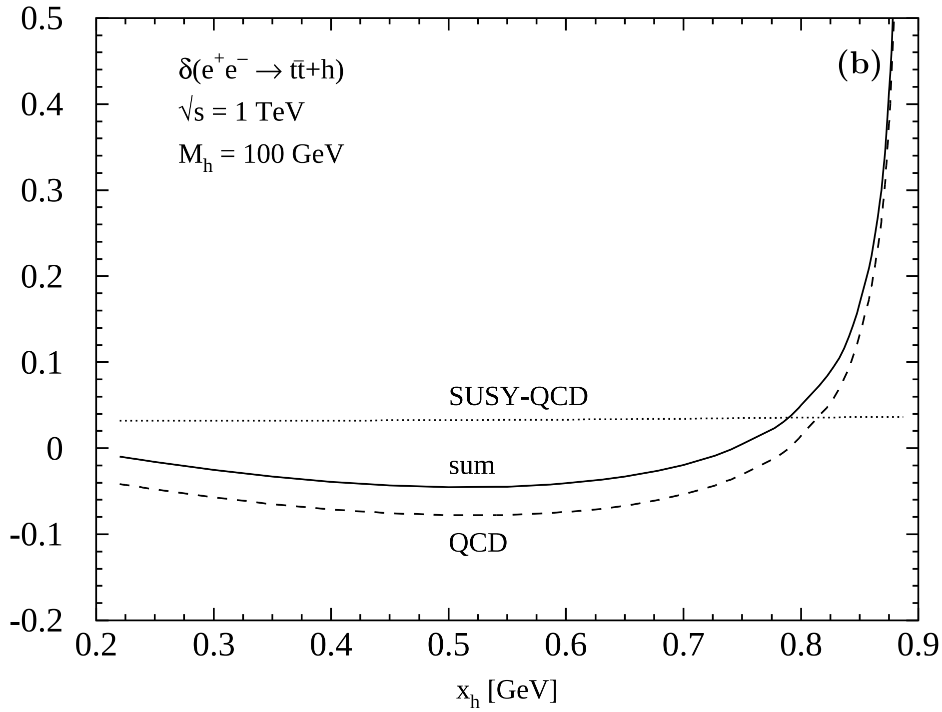


Figure 8: *(a) Higgs energy distribution of light scalar Higgs radiation off top quarks in e^+e^- collisions. The LO cross section is depicted by the dashed line and the full QCD- and SUSY-QCD corrected cross section by the full line; (b) Relative QCD, SUSY-QCD and total corrections to light scalar Higgs radiation off top quarks. The sharp rise of the QCD corrections towards $x_h \sim 0.9$ is induced by the Coulomb singularity.*

the other hand are dominated by the non-decoupling Δm_b terms, which can be absorbed and resummed in the corresponding bottom Yukawa couplings. We have shown that this absorption reduces the SUSY-QCD corrections to a moderate size, but still cancels the pure QCD corrections to a large extent in the resonant as well as continuum regimes.

Based on the size of the electroweak corrections to Standard Model $t\bar{t}H$ production the determination of the full SUSY-electroweak corrections to these processes turns out to be necessary, to reduce the overall theoretical uncertainties to a level which allows accurate measurements of the Higgs Yukawa couplings.

Acknowledgements. We would like to thank S. Dittmaier, M. Mühlleitner and P. Zerwas for valuable comments on the manuscript.

References

[1] P.W. Higgs, Phys. Lett. **12** (64) 132; F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321; G.S. Guralnik, C.R. Hagen and T.W. Kibble, Phys. Rev. Lett. **13** (1964) 585.

[2] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193** (1981) 150; N. Sakai, Z. Phys. **C11** (1981) 153; K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **67** (1982) 1889, *ibid.* **68** (1982) 927, (E) *ibid.* **70** (1983) 330, *ibid.* **71** (1984) 413; E. Witten, Nucl. Phys. **B231** (1984) 419.

[3] See e.g. G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. **C28** (2003) 133 [arXiv:hep-ph/0212020].

[4] H. Haber and G. Kane, Phys. Rep. **117** (1985) 75.

[5] A. Djouadi, J. Kalinowski, P. Ohmann and P.M. Zerwas, Z. Phys. **C74** (1997) 93 [arXiv:hep-ph/9605339].

[6] A. Djouadi, J. Kalinowski and P.M. Zerwas, Mod. Phys. Lett. **A7** (1992) 1765 and Z. Phys. **C54** (1992) 255.

[7] S. Dittmaier, M. Krämer, Y. Liao, M. Spira and P.M. Zerwas, Phys. Lett. **B441** (1998) 383 [arXiv:hep-ph/9808433].

[8] S. Dittmaier, M. Krämer, Y. Liao, M. Spira and P.M. Zerwas, Phys. Lett. **B478** (2000) 247 [arXiv:hep-ph/0002035].

[9] S. Dawson and L. Reina, Phys. Rev. **D57** (1998) 5851 [arXiv:hep-ph/9712400] and **D59** (1999) 054012 [arXiv:hep-ph/9808443].

[10] S. Dawson and L. Reina, Phys. Rev. **D60** (1999) 015003 [arXiv:hep-ph/9812488].

[11] G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Shimizu and Y. Yasui, Phys. Lett. **B571** (2003) 163 [arXiv:hep-ph/0307029]; A. Denner, S. Dittmaier, M. Roth and M. Weber, Phys. Lett. **B575** (2003) 290 [arXiv:hep-ph/0307193] and Nucl. Phys. **B680** (2004) 85 [arXiv:hep-ph/0309274]; Y. You, W.-G. Ma, H. Chen, R.-Y. Zhang, S. Yan-Bin and H.-S. Hou, Phys. Lett. **B571** (2003) 85 [arXiv:hep-ph/0306036].

[12] S.H. Zhu, arXiv:hep-ph/0212273.

[13] B. C. Allanach *et al.*, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, Eur. Phys. J. C **25** (2002) 113 [eConf **C010630** (2001) P125] [arXiv:hep-ph/0202233].

[14] M. Carena, M. Quirós and C.E.M. Wagner, Nucl. Phys. **B461** (1996) 407 [arXiv:hep-ph/9508343]; H.E. Haber, R. Hempfling and A.H. Hoang, Z. Phys. **C75** (1997) 539 [arXiv:hep-ph/9609331]; S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rev. **D58** (1998) 091701 [arXiv:hep-ph/9803277].

[15] A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. **108** (1998) 56 [arXiv:hep-ph/9704448].

[16] L. Hall, R. Rattazzi and U. Sarid, Phys. Rev. **D50** (1994) 7048 [arXiv:hep-ph/9306309]; R. Hempfling, Phys. Rev. **D49** (1994) 6168; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. **B426** (1994) 269 [arXiv:hep-ph/9402253]; D. Pierce, J. Bagger, K. Matchev and R. Zhang, Nucl. Phys. **B491** (1997) 3 [arXiv:hep-ph/9606211].

[17] H.E. Haber, M.J. Herrero, H.E. Logan, S. Penaranda, S. Rigolin and D. Temes, Phys. Rev. **D63** (2001) 055004 [arXiv:hep-ph/0007006].

[18] M. Carena, D. Garcia, U. Nierste and C.E.M. Wagner, Nucl. Phys. **B577** (2000) 88 [arXiv:hep-ph/9912516];

[19] J. Guasch, P. Häfliger and M. Spira, Phys. Rev. **D68** (2003) 115001 [arXiv:hep-ph/0305101].

[20] For a recent summary see J.F.Gunion, T.Han, J.Jiang and A.Sopczak, Contribution to the LHC/LC Physics Document, *eds.* G.Weiglein et al., arXiv:hep-ph/0410364.